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Summary: This research has resulted in several technical contributions in the areas of estimation and control for systems with unknown parameters. A technique was developed for direct synthesis of Kalman-Bucy type filters with low sensitivity to parameter variations [1], [2]. This technique is believed to be of general significance in the area of optimum estimation. Results obtained in the area of combined estimation and control, on the other hand, are less general in nature. Here, techniques for combined estimation and control were applied to specific problems [3], [4]. In [5], extensions were made to the sequential least squares estimation technique reported in [6] and [7]. These extensions are related to improved estimation and simplified estimators. Research has continued in this estimation technique, and at the present time, some interesting preliminary results are available on improved estimates in the presence of parameter uncertainties. These same results may be applicable to the combined estimation and control technique reported in [8].

Detailed technical reports have already been submitted on research related to combined estimation and control [3], [4], and on one aspect of the sequential estimation problem [5]. Therefore, these are not considered in any detail herein. It is too early to report on the preliminary results mentioned relating to sequential estimation. However, a detailed report will be forwarded when conclusive results are available. This should be before September, 1969. A detailed technical report relating to low sensitivity Kalman-Bucy type filters is in preparation and will be forwarded before August 1, 1969. Conclusive results for this work are already available, and these are outlined briefly below.

Discussion: Results obtained in low sensitivity Kalman-Bucy type filters pertain to the steady state design of a filter for estimating the state of a linear time invariant system from noisy measurements of the output when certain dynamical and/or statistical parameters were only partially known. It was assumed in this work that the vector of unknown parameters $\underline{\alpha}$ was constant and confined to a bounded domain A. If $\underline{\alpha}$ were known one would simply build the Kalman filter for that value of $\underline{\alpha}$. The major advantages of the Kalman filter, aside from its optimality, are that it is recursive and easily implemented. It is desirable therefore to retain these latter advantages in any optimally insensitive filter. Thus, lacking exact knowledge of $\underline{\alpha}$ a filter identical in form to that of the Kalman filter was selected. Now, however, the feedback and feedforward gains are adjusted in a manner to satisfy an appropriate sensitivity criterion. These adjustable gains, denoted here by the vector $\underline{\beta}$, belong to a bounded domain B of sufficient range and dimensionality to generate the Kalman filter for every value of $\underline{\alpha} \in A$.

The trace of the estimation error covariance matrix was chosen as the measure of filter error performance. It is well known that the Kalman filter minimizes the trace of this matrix. In general, this error measure is a function of both $\underline{\alpha}$ and $\underline{\beta}$, and is denoted here by $J(\underline{\alpha}, \underline{\beta})$. For a given $\underline{\alpha}$ the minimum value of $J(\underline{\alpha}, \underline{\beta})$ attained by the Kalman filter is $J_0(\underline{\alpha})$. Clearly

$$J(\underline{\alpha}, \underline{\beta}) \geq J_0(\underline{\alpha}) \qquad \forall\, \underline{\alpha} \in A, \quad \forall\, \underline{\beta} \in B$$

Since $\underline{\alpha}$ is unknown and $\underline{\beta}$ alone is available for selection by the designer, it seems most natural to view $\underline{\alpha}$ and $\underline{\beta}$ as adversaries in the game-theoretic sense. With this in mind, three sensitivity measures and their associated rules of synthesis seem appropriate. They are:

$$S_1 = \min_{\underline{\beta} \in B} \; \max_{\underline{\alpha} \in A} \; J(\underline{\alpha}, \underline{\beta})$$

$$S_2 = \min_{\underline{\beta} \in B} \; \max_{\underline{\alpha} \in A} \; \{J(\underline{\alpha}, \underline{\beta}) - J_0(\underline{\alpha})\}$$

$$S_3 = \min_{\underline{\beta} \in B} \; \max_{\underline{\alpha} \in A} \; \left\{\frac{J(\underline{\alpha}, \underline{\beta}) - J_0(\underline{\alpha})}{J_0(\underline{\alpha})}\right\}$$

The S_1 design simply minimizes the maximum value of J over the unknown parameter set $\underline{\alpha}$. This places an upper bound on the estimation error and might be interpreted as a "worst case" design. The second and third criteria minimize the maximum absolute and relative deviation respectively of the filter error from optimum over the unknown parameter set $\underline{\alpha}$. Thus the S_2 and S_3 criteria force the filter error to track the optimum error within some tolerance over the entire set $\underline{\alpha} \in A$. In each case the above procedures yield a fixed value of $\underline{\beta}$ and therefore a fixed filter design good for all values of $\underline{\alpha}$.

The primary research effort was directed toward the design of optimally insensitive filters in the presence of uncertain system and measurement noise statistics. Specifically, it was assumed that elements of the system and measurement noise covariance matrices, Q and R respectively, were unknown. These unknown elements then constitute the vector $\underline{\alpha}$.

In the case of the S_1 filter, several interesting results are available. First and foremost is that

$$S_1 = \min_{\underline{\beta} \in B} \max_{\underline{\alpha} \in A} J(\underline{\alpha}, \underline{\beta})$$

$$= \max_{\underline{\alpha} \in A} \left[\min_{\underline{\beta} \in B} J(\underline{\alpha}, \underline{\beta}) \right]$$

$$= \max_{\underline{\alpha} \in A} J_0(\underline{\alpha})$$

since by definition

$$\min_{\underline{\beta} \in B} J(\underline{\alpha}, \underline{\beta}) = J_0(\underline{\alpha})$$

This result is crucial because it replaces the very difficult min-max problem with a relatively simple maximization of the optimal returns over all unknown $\underline{\alpha} \in A$.

The above result implies that the S_1 filter is unique, a highly desirable result. It has also been shown that $J_0(\underline{\alpha})$ is concave in A and has continuous first derivatives everywhere in A. Thus relatively straightforward maximizing algorithms such as "steepest ascent" techniques will yield the maximizing value of $\underline{\alpha}$. The vector $\underline{\beta}$ may then be computed from $\underline{\alpha}$ yielding the S_1 filter.

In the case of S_2 and S_3 filters, equally gratifying theoretical results are available. First of all, the S_2 and S_3 filters are known to be unique. Furthermore, it has been shown that the value of $\underline{\beta}$ yielding the min-max is actually optimal for at least one $\underline{\alpha} \in A$. This is intuitively appealing and lends validity to the empirical approach commonly used in practice. Unfortunately, min-max does not equal max-min for the S_2 and S_3 filters and one is thus forced to solve the complete min-max problem. It has been shown, however, that the maximum of S_2 and S_3 is attained over a finite set of points in A called the extreme points of A, thereby greatly reducing the search problem in that domain. Several interesting and significant examples employing this technique are worked out in [2].

<u>Grant Activities</u>: This grant has supported two faculty members, the co-principal investigators, and five graduate students. Three of these students have received Master's degrees and did thesis work related to the grant research topic. One of these would have continued on for Ph.D. work, but was drafted. A fourth student is doing his Master's thesis now and should be completed by February, 1970. The fifth graduate student, J. A. D'Appolito, is a Ph.D. candidate and will complete the degree requirements by September, 1969.

One publication has already resulted from the grant research (reference [1]), and it is expected that several others will be forthcoming, especially from the work in [2], and also from the current work in sequential estimation.

This grant has stimulated the graduate research program at the University of Massachusetts and led to several significant results.

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